Top Student Errors in AP Calculus

1. $f''(x) = 0 \Leftrightarrow (x, f(x))$ **is a point of inflection**.

Example: 1969 AB/BC 17

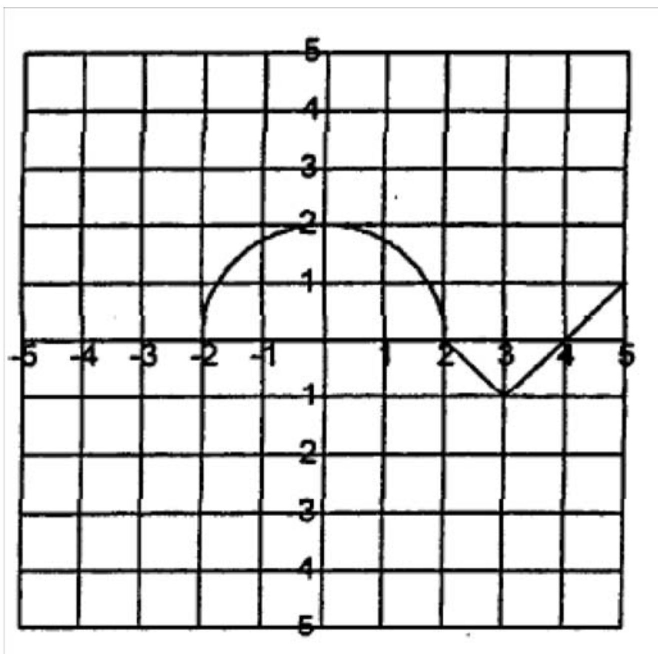
The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) (0,0) only **(B) (3,162) only** (C) (4,256) only

(D) (0,0) and (3,162) (E) (0,0) and (4,256)

In 1969 62% of the AB students chose choice (D) and 18% chose choice (B).

1997 AB/BC 5 : The graph of a function *f* consists of a semicircle and 2 line segments as shown below. Let *g* be the function given by $g(x) = \int_0^x f(t)dt$.



Part (D) Find the x-coordinate of each point of inflection of the graph of *g* on the open interval (-2,5). Justify your answer.

Adopted from Dan Kennedy's Top Ten Errors on the AP Test

2. ***f(x)* is a maximum (minimum) <=> $f'(x)=0$.**

Example: 1993 AB 19

Let *f* be the function defined by $f(x)=\begin{cases} x^3, \text{ for } x \le 0 \\ x, \text{ for } x > 0 \end{cases}$

(A) *f* is an odd function.
(B) *f* is discontinuous at x = 0.
(C) *f* has a relative maximum.
(D) $f'(0)=0$.
(E) $f'(x)>0 \text{ for } x \ne 0$.

22% of the students chose (A), 14% chose (D) and 33% chose (E). 11% of the students omitted the answer.

Example: 1998 AB 89

If g is a differentiable function such that *g(x)<0* for all real numbers *x* and if $f'(x)=\left(x^2-4\right)g(x)$, which of the following is true?

(A) *f* has a relative maximum at x = -2 and a relative minimum at x = 2.
(B) *f* has a relative minimum at x = -2 and a relative maximum at x = 2.

(C) *f* has relative minima at x = -2 and at x = 2.

(D) *f* has relative maxima at x = -2 and at x = 2.

(E) It cannot be determined if *f* has any relative extrema.

38% of the AB students answered the question correctly in 1998.

3. **Average rate of change of *f* on [a,b] is** $\frac{f'(a)+f'(b)}{2}$ **.**

This, of course, is the wrong definition of ARC. The ARC of *f* on [a,b] is $\frac{f(b)-f(a)}{b-a}$.

Example: 1998 AB 3 (B)

A table for values for the velocity;, *v(t),* in *ft/sec* of a car traveling on a straight road, for $0 \le t \le 50$.

t seconds	0	5	10	15	20	25	30	35	40	45	50
v(t) ft/sec	0	12	20	30	55	70	78	81	75	60	72

Answer: $\frac{f(50)-f(0)}{50-0} = \frac{72-0}{50} = \frac{72}{50} ft/sec^2$

4. **Volume by washers is** $\int_a^b \pi\left(R-r\right)^2 dx$ **.**

Example: Let R be the region between the graphs of y = 1 and y = sin x from x = 0 to x = $\frac{\pi}{2}$. The volume obtained by revolving R about the x-axis is given by

(A) $2\pi\int_0^{\frac{\pi}{2}} x\sin x dx$ (B) $2\pi\int_0^{\frac{\pi}{2}} x\cos x dx$ (C) $\pi\int_0^{\frac{\pi}{2}}\left(1-\sin x\right)^2 dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \sin^2 x dx$ (E) $\pi\int_0^{\frac{\pi}{2}}\left(1-\sin^2 x\right)dx$

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5. **Separable differential equations can be solved without separating the variables.**

Example: 2008 AB 23 Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition y(3) = -2?

(A) $y = 2e^{-9+\frac{x^3}{3}}$ (B) $y = -2e^{-9+\frac{x^3}{3}}$ (C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$ (E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

23% got this problem correct in 2008.

6. $\frac{d}{dx} f(y) = f'(y)$ **and other chain rule errors.**

Example: 2008 AB 4 If $f(x) = (x-1)(X^2+2)^3$, then f ‘ (x) =

(A) $6x(x^2+2)^2$ (B) $6x(x-1)(x^2+2)^2$ (C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$ (E) $-3(x-1)(x^2+2)^2$

39% of the students got the right answer in 2008.

7. **Graders will assume the correct antecedents for all pronouns used in justification. [My students were never permitt4ed to use the word “it” in justification. They had to talk about the function, the first derivative and the second derivative.]**

8. **Only use the calculator for 4 operations:**

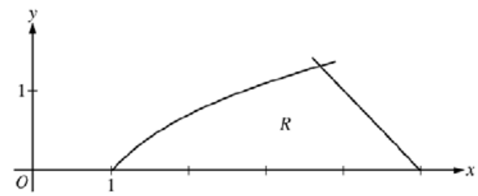
a) graphing in an arbitrary viewing window

(2007 AB/BC1) Let R be the region in the first and second quadrants bounded above by $y = \dfrac{20}{1+x^2}$ and below by the horizontal line y = 2.

b) evaluating a definite integral

2012 AB/BC2 (A)

Find the area of R.



c) evaluating a derivative at a point (the numerical derivative in a calculator that is not a CAS)

2008 #82 A particle moves along a straight line with velocity given by $v(t) = 7 - \left(1.01\right)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time t=3?

(A) -0.914 (B) 0.005 (C) 5.486 (D) 6.086 (E) 18.087

d) find the zeros of a function or a point of intersection

2008 #83 What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$.

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

61% of the students got this problem correct.

9. **Universal logarithmic antidifferentiation:**

$$\int \frac{1}{f(x)}dx = \ln\left|f(x)\right| + C$$

Example: $\displaystyle\int \frac{4x}{1+4x^4}dx$

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10. **Finding Derivatives on Inverse Functions**

Example 2008 AB 28 Let *f* be a diffentiable function such that *f(3)=15, f(6)=3, f ' (3) = -8,* and *f '(6)= -2.* The function *g* is differentiable and $g(x) = f^{-1}(x)$ for all *x.* What is the value of *g'(3)*?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

(E) The value of *g'(3)* cannot be determined from the information given.

14% of the students got this problem correct in 2008.

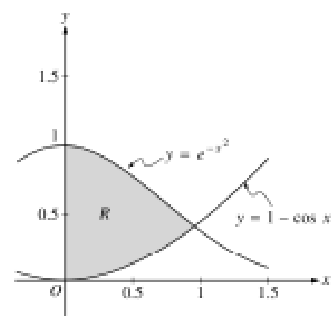
Example 2 2008 AB 26 What is the slope of the line tangent to the curve y=arctan(4x) at the point at which x = 1/4?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

25% of the students got this problem correct in 2008.

11. **Omission of units in the solution of a "real life" calculus problem.**

12. **Rounding too early when solving a problem.**



Example: 2000 AB/BC 1 Let R be the region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$ and $y = 1 - \cos x$. Find the area of R, etc.

The curves intersect at x=0.941944. Store 0.941944 in your calculator as B. Define B for the reader. Then express your answer as a definite integral.

13. **Forgetting the "C" in an analytic solution of a differential equation. I would always tell my students that if they forgot the "C" they should do another problem.**

Example: 2000 AB 6 Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

A) Find a solution y=f(x) to the differential equation satisfying $f(0) = \frac{1}{2}$.

B) Find the domain and range of the function *f* found in part A.

Solution:

1 point	$e^{2y}dy = 3x^2dx$	separation of variables
1 point	$\frac{1}{2}e^{2y}$	antiderivative
1 point	x^3	antiderivative
1 point	$\frac{1}{2}e^{2y} = x^3 + C$	If the student omitted the C the student obtained a maximum 3 of 6 points.
1 point	$\frac{1}{2}e^{2\cdot\left(\frac{1}{2}\right)} = (0)^3 + C$ $C = \frac{1}{2}e$	Uses the initial condition
1 point	$\frac{1}{2}e^{2y} = x^3 + \frac{1}{2}e$ $\frac{1}{2}e^{2y} = \frac{2x^3 + e}{2}$ $e^{2y} = 2x^3 + e$ $y = \frac{1}{2}\ln\left(2x^3 + e\right)$	Solves for y
1 point	$x > -\left(\frac{1}{2}e\right)^{\frac{1}{3}}$	domain
1 point	$-\infty < y < \infty$	range

14. **Reading and interpreting tabular information**

Example: 2008 AB/BC 14

x	0	1	2	3
f'(x)	5	0	-7	4

The polynomial function *f* has selected values of its second derivative *f"* given in the table above. Which of the following statements must be true?

(A) *f* is increasing on the interval (0,2).
(B) *f* is decreasing on the interval (0,2)
(C) *f* has a local maximum at x = 1.
(D) The graph of *f* has a point of inflection at x = 1.
(E) The graph if *f* changes concavity in the interval (0,2).

29% of the AB students got this problem correct and 45% of the BC students got this problem correct in 2008.

15. **Bad Algebra Errors**

You can probably name a few common algebra errors you have seen your students make