2007 Form B AB1

A. Area of R $= \int_{0.44605703}^{1.55394} \left(e^{2x-x^2} - 2\right)dx = 0.514$

B.

Area of S = $\int_{0}^{2} \left(e^{2x-x^2} - 1\right)dx$ – Area of A

$= 2.060156939 - .5141427856$

$= 1.546$

C. $\int_{.44605703}^{1.55394} \left(\pi\left(e^{2x-x^2}\right)^2 - \pi(1)^2\right)dx$

2007 Form B AB2

A. $Acceleration = v'(t) = \left.\frac{d(\sin(t^2))}{dt}\right|_{t=3} = -5.467 \ or \ -5.466$

B. $\text{Total Distance} = \int_0^3 \left|\sin(t^2)\right| dt = 1.702$

C. $\text{Position} = 5 + \int_0^3 \sin(t^2)\,dt = 5.773 \text{ or } 5.774$

D. Particle comes to a stop and changes direction at t = 1.772, 2.5066, 3.06998, 3.544907

Using the equation in part C and evaluating the position at these times yields the following positions

$$\text{Position}(1.772) = 5 + \int_0^{1.772} \sin(t^2)\,dt = 5.894$$

$$\text{Position}(2.5066) = 5 + \int_0^{2.5066} \sin(t^2)\,dt = 5.430$$

$$\text{Position}(3.06998) = 5 + \int_0^{3.06998} \sin(t^2)\,dt = 5.788$$

$$\text{Position}(3.5449077) = 5 + \int_0^{3.5449077} \sin(t^2)\,dt = 5.486$$

The particle is further to the right at t=1.772.

This can also be verified by the area between the function v(t) and the t axis. Between 0<t<1.772 (portion 1)the particle is moving to the right. Then between 1.772<t<2.5066 (portion 2) the particle moves to the left but not as much as it had to the right (in portion 1) because there is less area between the axis and the function in portion 2 than portion 1. Again between 2.5066<t<3.0699 (portion 3) the particle moves to the right but again not as much as the previous portion because there is less area under the third portion than in the second portion. Between 3.0699<t<3.544 (portion 4) the particle again moves left but not as much as it had in third portion because there is less area in portion 4. Then finally between $3.5449 < t < \sqrt{5\pi}$ the particle moves to the right, but not as far to the right in this portion as it had moved left in the last portion. Therefore the furthest to the right is at time 1.772.

2007 Form B AB3

A. $w'(20) = -0.285 \text{ or } -0.286$. This shows that the wind chill temperature is decreasing at a rate of 0.285 degrees F/mph.

B.

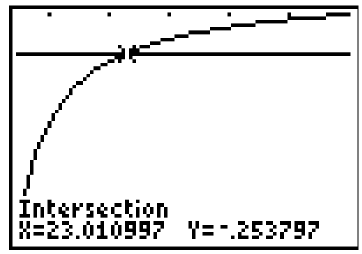
$$w_{avg} = \frac{w(60) - w(5)}{60 - 5} =$$

$$-0.253797 \text{ or } -0.253 \text{ or } -0.254$$

$$w'(v) = 22.1(0.16)v^{-.084}$$

$$-0.253 = 22.1(0.16)v^{-.084}$$

Graphically this solution is at v = 23.011 miles per hour.



C.

$$\frac{dw}{dv} = (22.1)(0.16)v^{-0.84}, \frac{dv}{dt} = 5$$

$$v = 20 + 5t$$

$$\frac{dw}{dt} = \frac{dw}{dv} \bullet \frac{dv}{dt}$$

$$\frac{dw}{dt} = 3.536v^{-0.84} \bullet 5 = 17.68v^{-0.84} = 17.68(20 + 5t)^{-0.84}\Big|_{t=3}$$

$$= 0.892 \ {}^oF / hour$$

Or

$$w(v) = 55.6 - 22.1v^{0.16}$$

$$v = 20 + 5t$$

$$w(t) = 55.6 - 22.1(20 + 5t)^{0.16}$$

$$w'(t) = (22.1)(0.16)(20 + 5t)^{-0.84} \bullet 5$$

$$w'(t) = 17.68(20 + 5t)^{-0.84}$$

$$w'(3) = 17.68(20 + 5 \bullet 3)^{-0.84} = 17.68(35)^{-0.84} \ {}^oF / hour$$

$$= 0.892 \ {}^oF / hr$$

2007 Form B AB4

Information about f ‘ and f “.

x	-5<x<-3	-3<x<1	1<x<4	4<x<5
f ’	positive	negative	positive	Negative
f	increasing	decreasing	increasing	decreasing

x	-5<x<-4	-4<x<-3	-3<x<-1	-1<x<1	a<x<2	2<x<5
f “	positive	negative	negative	positive	positive	negative
f	ccu	ccd	ccd	ccu	ccu	ccd

A.

The function f has a relative maximum at x = -3 and at x = 4 because it is at x = -3 that the function changes from increasing to decreasing (because the derivative of f changes from positive to negative.)

B. There are three points of inflection. The occur at x = -4, -1, and 2. This is because the second derivative of f changes sign at these locations.

C. The function f is concave up and has positive slope for -5<x<-4 and 1<x<2 because f “ has a positive value and f ‘ has a positive value.

D. There are two candidates for the absolute minimum. They are x = -5 and +1. We know that f(1) = 3. Starting at x = -5 the function f increases until x = -3 by a value equal to the area of the semi-circle with radius 1 or π. Then the function f decrease between -3<x<-1. The decrease in the value of the function equal the area of the semi-circle with a radius of 2 or 2π. (After x>1 the function increases so there are no other candidates for the absolute minimum.) Therefore the absolute minimum is at x = 1

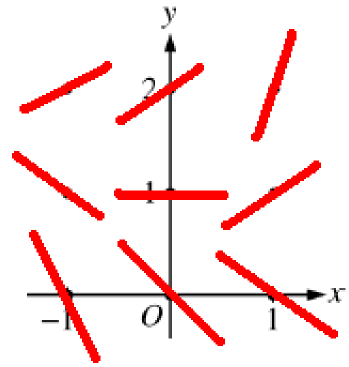
Summary : f (-5) = C, f (-3) = $C+\pi$, f(1) = $C-\pi$. Therefore the absolute minimum occurs at x = 1.

Alternately: We know that f(1)=3. By the accumulation function we know that $f(-3)=3+2\pi$ and $f(-5)=3-\frac{\pi}{2}+2\pi=3+\frac{3}{2}\pi$. Therefore the minimum value of the function is 3.

2007 Form B AB5

A.

coordinate	slope
(1,0)	- ½
(1,1)	½
(1,2)	-1
(0,1)	0
(-1,0)	- 3/2
(-1,1)	½
(-1,2)	½



B.

$$\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx}$$
$$= \frac{1}{2} + \frac{1}{2}x + y - 1$$
$$= -\frac{1}{2} + \frac{1}{2}x + y$$

For f to be concave up the second derivative of f must be positive therefore

$$\frac{d^2y}{dx^2} = -\frac{1}{2} + \frac{1}{2}x + y > 0$$
$$y > \frac{1}{2} - \frac{1}{2}x$$

This describes a half plane above the line

$$y = \frac{1}{2} - \frac{1}{2}x$$

Alternatively: Since we know that the point (0,1) is in this half plane so the second derivative at (0,1) is positive. We can also see that at this point $\frac{dy}{dx} = 0$. So this tells us that there is a minimum at (0,1).

C. We know that f(0)=1 We know to the left of this point the function is decreasing and to the right of this point the function is increasing (based upon the slopefield.) Therefore the function f has a relative minimum at the point (0,1).

D. If y = mx + b then $\frac{dy}{dx} = m$.

Substituting these in the differential equation yields:

$$m = \frac{1}{2}x + (mx + b) - 1$$
$$\left(m + \frac{1}{2}\right)x + \left(b - m - 1\right) = 0$$

For this to be true

$$m + \frac{1}{2} = 0 \;\; and \;\; b - m - 1 = 0$$
$$m = -\frac{1}{2} \;\; and \;\; b + \frac{1}{2} - 1 = 0 \; or \; b = \frac{1}{2}$$

Alternatively:

$$\frac{d^2y}{dx^2} = -\frac{1}{2} + \frac{1}{2}x + y = 0$$
$$y = \frac{1}{2} - \frac{1}{2}x$$

This says m = -½ and b = ½ .

2007 Form B AB6

A. $f_{avg} = \dfrac{f(5) - f(2)}{5 - 2} = \dfrac{2 - 5}{5 - 2} = -1$

Since the average rate of change of f between x = 2 and x = 5 is -1, by the MVT there must be a location c such that 2<c<5 where $f_{avg} = f'(c) = -1$.

B.

$$g'(x) = f'(f(x)) \bullet f'(x) \qquad g'(x) = f'(f(x)) \bullet f'(x)$$

$$g'(2) = f'(f(2)) \bullet f'(2) \quad \text{and} \quad g'(5) = f'(f(5)) \bullet f'(5)$$

$$= f'(5) \bullet f'(2) \qquad = f'(2) \bullet f'(5)$$

Since g'(2)=g'(5), then $\dfrac{g'(2) - g'(5)}{2 - 5} = g'_{avg} = 0$. By the MVT there must exist a 2<c<5 such that $g'_{avg} = g''(c) = 0$.

C.

$$g''(x) = f'(f(x)) \bullet f''(x) + f'(x) \bullet f''(f(x)) f'(x)$$

$$= f'(f(x) \bullet f''(x) + (f'(x))^2 \bullet f''(f(x))$$

$$= f'(f(x) \bullet 0 + (f'(x))^2 \bullet 0$$

$$= 0$$

For g to have a point of inflection g" must change sign. But since we know that g" equals zero it cannot change sign. Therefore these is no point of inflection.

D.

$$h(5) = f(5) - 5 = 2 - 5 = -3$$

$$h(2) = f(2) - 2 = 5 - 2 = 3$$

Therefore, by the IVT we know that there exists a 2<r<5 such that h(r)=0.

A.

$$speed = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\arctan\left(\frac{t}{1+t}\right)\right)^2 + \left(\ln(t^2+1)\right)^2}\Bigg|_{t=4}$$

$$= 2.912$$

B.

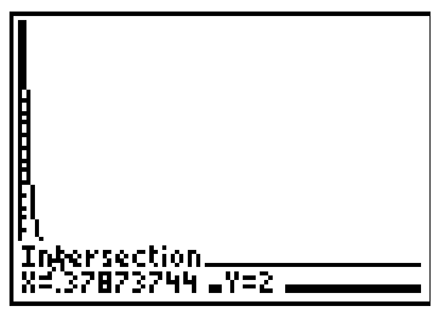
$$\text{Distance traveled} = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_0^4 \sqrt{\left(\arctan\left(\frac{t}{1+t}\right)\right)^2 + \left(\ln(t^2+1)\right)^2}$$

$$= 11.649 \text{ or } 11.650$$

C. $x(4) = -3 + \int_0^4 \arctan\left(\frac{t}{1+t}\right)dt = -.301$

D. Slope of the tangent line is $\dfrac{dy/dt}{dx/dt} = 2$



This occurs at t=.378

$$\left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle\Bigg|_{t=4} = \langle .0243, .470 \rangle$$

2007 Form B BC5

A.

$$\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx}$$
$$= 3 + 2(3x + 2y + 1)$$
$$= 3 + 6x + 4y + 2$$
$$= 5 + 6x + 4y$$

B.

$$\frac{dy}{dx} = m + re^{rx}$$
$$m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1$$
$$(3 + 2m)x + (2b - 1 - m) + (2 - r)e^{rx} = 0$$
$$3 + 2m = 0,\ 2b - 1 - m = 0,\ 2 - r = 0$$
$$m = -\frac{3}{2},\ r = 2,\ b = -\frac{1}{4}$$

C. Approximate value of $f(1) = \frac{9}{4}$

(0,2)	$\frac{dy}{dx} = 5$	y=5(x-0)-2	$\left(\frac{1}{2}, \frac{1}{2}\right)$
$\left(\frac{1}{2}, \frac{1}{2}\right)$	$\frac{dy}{dx} = \frac{7}{2}$	$y = \frac{7}{2}\left(x - \frac{1}{2}\right) + \frac{1}{2}$	$\left(1, \frac{9}{4}\right)$

D.

(0,k)	$\frac{dy}{dx} = 2k + 1$	$y = (2k + 1)(x) + k$ $y = 2k + \frac{1}{2}$	$\left(\frac{1}{2}, 2k + \frac{1}{2}\right)$
$\left(\frac{1}{2}, 2k + \frac{1}{2}\right)$	$\frac{dy}{dx} = \frac{7}{2} + 4k$	$y = \left(\frac{7}{2} + 4k\right)\left(x - \frac{1}{2}\right) + 2k + \frac{1}{2}$	$\left(1, \frac{9}{4} + 4k\right)$

A.

$$6-2x+\frac{1}{3}x^2-\frac{1}{27}x^3 \qquad \textit{general term}=\frac{6\left(-\frac{1}{3}\right)^k x^k}{k!}$$

B.

$$g(x)\approx\int 6-2x+\frac{1}{3}x^2-\frac{1}{27}x^3$$

$$g(x)\approx 6x-x^2+\frac{1}{9}x^3-\frac{1}{108}x^4$$

$$\text{general term}=\frac{6\left(-\frac{1}{3}\right)^{k+1} x^{k+1}}{(k+1)!}$$

C.

$$f(x)\approx 6-2x+\frac{1}{3}x^2-\frac{1}{27}x^3$$

$$f'(x)\approx -2+\frac{2}{3}x-\frac{1}{9}x^2$$

$$f'(ax)\approx -2+\frac{2ax}{3}-\frac{1}{3}(ax)^2$$

$$kf'(ax)\approx -2k+\frac{2akx}{3}-\frac{k}{3}(ax)^2$$

$$h(x)=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\ldots$$

$$-2k=1\Rightarrow k=-\frac{1}{2}$$

This implies that

$$\frac{2ka}{3}=1\Rightarrow\frac{2\left(-\frac{1}{2}\right)a}{3}=1\Rightarrow a=-3$$