

2007 Form B AB4

Information about f' and f'' .

x	$-5 < x < -3$	$-3 < x < 1$	$1 < x < 4$	$4 < x < 5$
f'	positive	negative	positive	Negative
f	increasing	decreasing	increasing	decreasing

x	$-5 < x < -4$	$-4 < x < -3$	$-3 < x < -1$	$-1 < x < 1$	$1 < x < 2$	$2 < x < 5$
f''	positive	negative	negative	positive	positive	negative
f	ccu	ccd	ccd	ccu	ccu	ccd

A.

The function f has a relative maximum at $x = -3$ and at $x = 4$ because it is at $x = -3$ that the function changes from increasing to decreasing (because the derivative of f changes from positive to negative.)

B. There are three points of inflection. They occur at $x = -4, -1, \text{ and } 2$. This is because the second derivative of f changes sign at these locations.

C. The function f is concave up and has positive slope for $-5 < x < -4$ and $1 < x < 2$ because f'' has a positive value and f' has a positive value.

D. There are two candidates for the absolute minimum. They are $x = -5$ and $x = 1$. We know that $f(1) = 3$. Starting at $x = -5$ the function f increases until $x = -3$ by a value equal to the area of the semi-circle with radius 1 or π . Then the function f decreases between $-3 < x < -1$. The decrease in the value of the function equals the area of the semi-circle with a radius of 2 or 2π . (After $x > 1$ the function increases so there are no other candidates for the absolute minimum.) Therefore the absolute minimum is at $x = 1$.

Summary : $f(-5) = C$, $f(-3) = C + \pi$, $f(1) = C - \pi$. Therefore the absolute minimum occurs at $x = 1$.

Alternately: We know that $f(1) = 3$. By the accumulation function we know that $f(-3) = 3 + 2\pi$ and $f(-5) = 3 - \frac{\pi}{2} + 2\pi = 3 + \frac{3}{2}\pi$. Therefore the minimum value of the function is 3.