2007 BC4

A. f(e)=2 and $f'(e)=e^2 \ln e = e^2$ so the tangent line at (e,2) is $y = e^2(x-e)+2$.

B.

$$f''(x) = x^2 \bullet \frac{1}{x} + (\ln x)(2x)$$

$$= x + 2x\ln x$$

So for 1<x<3, f "(x) is positive indicating that f is concave up.

C.

$$f(x) = \int x^2 \ln x dx$$

$$u = \ln x \qquad dv = x^2 dx$$

$$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$$

So

$$f(x) = \int x^2 \ln x dx$$

$$= (\ln x)\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3} \bullet \frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + c$$

$$f(e) = \frac{e^3 \ln e}{3} - \frac{e^3}{9} + c = 2 \Rightarrow c = 2 - \frac{2e^3}{9}$$

$$f(x) = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + 2 - \frac{2e^3}{9}$$