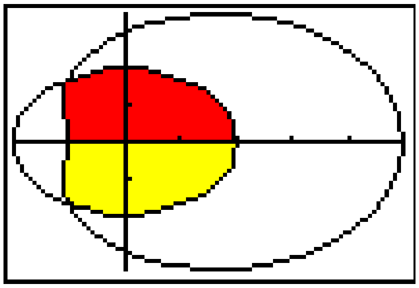
2007 BC3



A. Using symmetry of the red and yellow area the Area of R =

$$2\bullet\frac{1}{2}\int_0^{2\pi/3} 2^2 d\theta + 2\bullet\frac{1}{2}\int_{2\pi/3}^{\pi}(3+2\cos\theta)^2 d\theta = 10.370$$

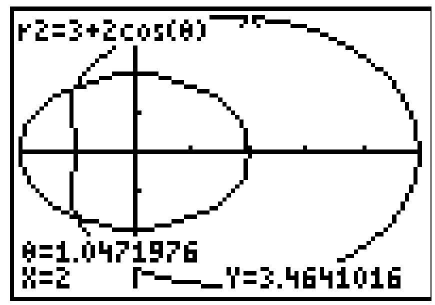
B.

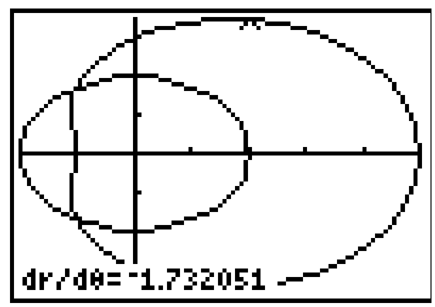
$$\frac{dr}{dt}=\frac{dr}{d\theta}=\frac{d(3+2\cos\theta)}{d\theta}=-2\sin\theta\Big|_{\theta=\frac{\pi}{3}}=-2\bullet\frac{\sqrt{3}}{2}=-\sqrt{3}$$

At the time when $\theta=\dfrac{\pi}{3}$ the particle is at a position

$$\left(\left(3+2\cos\frac{\pi}{3}\right)\cos\frac{\pi}{3},\left(3+2\cos\frac{\pi}{3}\right)\sin\frac{\pi}{3}\right)$$

$$\left(2,2\sqrt{3}\right)$$





and tracing out the curve in the polar coordinate axes.

$\dfrac{dr}{d\theta}$ Is equal to the rate at which the directed distance from the origin to the point is changing with respect to θ. So at this point the distance is decreasing at a rate of $\sqrt{3}\ units/radian$

C

$$\frac{dy}{dt}=\frac{dy}{d\theta}=\frac{d((3+2\cos\theta)\sin\theta)}{d\theta}$$

$$=-2\sin\theta\sin\theta+\cos\theta(3+2\cos\theta)$$

$$=-2\sin^2\theta+3\cos\theta+2\cos^2\theta\Big|_{\theta=\frac{\pi}{3}}$$

$$=-2\sin^2\frac{\pi}{3}+3\cos\frac{\pi}{3}+2\cos^2\frac{\pi}{3}$$

$$=-2\bullet\left(\frac{\sqrt{3}}{2}\right)^2+3\bullet\frac{1}{2}+2\left(\frac{1}{2}\right)^2$$

$$=-\frac{3}{2}+\frac{3}{2}+\frac{1}{2}=\frac{1}{2}$$

The particle is located at $\left(2,2\sqrt{3}\right)$ at the instant $\theta=\dfrac{\pi}{3}$ and the vertical distance is increasing at the rate of ½ unit per radian.