2007 AB4

A. $$x(t) = e^{-t}\sin t$$
$$x'(t) = e^{-t}\cos t - e^{-t}\sin t = e^{-t}(\cos t - \sin t)$$

$x(t) = 0$ where $\sin t = \cos t$ or $t = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

x'(t) is negative for $\dfrac{\pi}{4} < t < \dfrac{5\pi}{4}$ and positive for $0 < t < \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4} < t < 2\pi$. When x'(t) is negative then the particle is moving to the left. When x'(t) is positive the particle is moving to the right. There are two candidates for the location where the particle has reached the farthest to the left: at the start of the problem when the particle begins to move to the right or when $t = \dfrac{5\pi}{4}$ when the particle has stopped going to the left and begins to move to the right. X(0) = 0 and $x\left(\dfrac{5\pi}{4}\right) = -e^{\frac{-5\pi}{4}} \bullet \dfrac{\sqrt{2}}{2} < 0$. So it is farthest to the left at $t = \dfrac{5\pi}{4}$.

$$x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$$
B. $$= -e^{-t}\cos t + e^{-t}\sin t - e^{-t}\sin t - e^{-t}\cos t$$
$$= -2e^{-t}\cos t$$

so Ax"(t) + x'(t) + x(t)

$$Ax''(t)+x'(t)+x(t) = A(-2e^{-t}\cos t)+e^{-t}\cos t - e^{-t}\sin t + e^{-t}\sin t = 0$$

$$e^{-t}\cos t(-2A+1) = 0$$

$$A = \frac{1}{2}$$