

A.

$$\text{Gallons entering tank} = \int_0^7 f(t) dt = 8,263.806 \text{ or } 8,263.807$$

or rounded to the nearest gal: 8,264 gallons

B. Since  $f(t)$  and  $g(t)$  represent the rates at which water is entering and leaving the tank, respectively, then

$$f(t) - g(t) = \begin{cases} f(t) - 250 & \text{when } 0 < t < 3 \\ f(t) - 2000 & \text{when } 3 < t < 7 \end{cases} \quad \text{We know}$$

that  $f(t) - g(t) = 0$  when  $t = 1.617$  or  $t = 5.076$

The amount of water in the tank is decreasing when  $f(t) < g(t)$ . This occurs when  $0 < t < 1.617$  or when  $3 < t < 5.076$ .

Alternatively: *Based on the graph of  $f$  and  $g$  we know until  $t = 1.617$  the rate of water leaving the tank is more than the rate at which water is being added to the tank. Therefore the amount of water will be decreasing. After  $t = 1.617$  and before  $t = 3$  the rate of water entering the tank is more than the rate at which the water is leaving therefore the amount of water is increasing. After  $t = 3$  until  $t = 5.076$  the rate at which water is entering is less than the rate at which the water is leaving. This means the amount of water is decreasing. Then from  $t = 5.076$  to  $t = 7$  the rate at which water is entering is more than the rate at which it is leaving so the amount of water is increasing.*

C. Since  $f(t)$  is continuous on a closed interval and we are looking for the absolute maximum value we need to usually only look at the amount of water in the tank at the endpoints and at all critical values. At the two critical points there could not be a maximum because at both of these locations a relative minimum exists. We need to check only the amount of water at:  $t = 0$ ,  $t = 3$ , and  $t = 7$ .

$t = 0$	$t = 3$	$t = 7$
5000 gallons	$5000 + \int_0^3 (f(t) - g(t)) dt$ $= 5126.590 \text{ or } 5127 \text{ gallons}$	$5000 + \int_0^3 (f(t) - g(t)) dt + \int_3^7 (f(t) - g(t)) dt$ $= 4514 \text{ gallons}$

From the chart of values it is clear that the absolute maximum occurs at  $t = 3$  when 5127 gallons are in the tank.