

2003 AB2

2a. Acceleration is the derivative of velocity so

$$\left. \frac{dy}{dt} = \frac{d\left(- (t+1) \sin\left(\frac{t^2}{2}\right)\right)}{dt} \right|_{t=2} = 1.58758$$

$v(2) < 0$. So since acceleration is positive at $t = 2$ when $v(2) < 0$. The speed is decreasing at $t = 2$.

2b. Particle changes direction when $v(t) = 0$. Solving this graphically gives $t = 2.50663$. A sign study of $v(t)$ indicates that $v(t)$ is negative from $(0, 2.50663)$ and positive from $(2.50663, 3)$.

2c. Total distance

$$\int_0^{2.50663} -v(t) dt + \int_{2.50663}^3 v(t) dt = 4.33382$$

2d. This is the position function. There are three possibilities for the maximum distance from the origin. At the start of the problem when $t = 0$, the particle is at position $x = 1$. When the particle turns around at $t = 2.50663$ (or $\sqrt{2\pi}$) the particle has traveled 3.265483 to the left and ended up at position $x = -2.265483$ (to the left of the origin). Then the particle turns around and travels to the right 1.0683354 to a position of -1.197 which is still to the left of the origin at $t = 3$. Find the position at each of these times yields the following:

t	x(t)
0	1
2.50663	-2.26548
3	-1.197

The maximum distance from the origin is -2.265 at $t = \sqrt{2\pi}$.