2003 BC6

6a. $f'(x) = \frac{-2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \ldots \frac{(-1)^n (2nx^{2n-1})}{(2n+1)!}$

$f'(0) = 0$

$$f''(x) = \frac{-2}{3!} + \frac{4 \cdot 3x^2}{5!} - \frac{6 \cdot 5x^4}{7!} + \ldots \frac{(-1)^n (2n \cdot (2n-1) x^{2n-2})}{(2n+1)!}$$

$f''(0) = -\frac{1}{3}$

Since $f''(0) = -\frac{1}{3}$ we can use the second derivative test and determine that since the first derivative is equal to zero at zero and the second derivative is negative the function f has a maximum at x = 0.

f '(0) = 0 so let's use the first derivative test.

When x < 0 f ' (x) < 0. When x > 0 f '(x) > 0. So since the sign of f ' changes from negative to positive there must be a local maximum at x = 0.

6b. $f(1) \approx 1 - \frac{1}{3!}$ for a two term approximation of f(1). So the most error will be the next term. $\frac{1}{5!} = \frac{1}{120} < \frac{1}{100}$

6c

$$f'(x) = \frac{-2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \ldots \frac{(-1)^n (2nx^{2n-1})}{(2n+1)!}$$

$$xf'(x) = \frac{-2x^2}{3!} + \frac{4x^4}{5!} - \frac{6x^6}{7!} + \ldots \frac{(-1)^n (2nx^{2n})}{(2n+1)!}$$

$$f'(x) = \frac{-2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \ldots \frac{(-1)^n (2nx^{2n-1})}{(2n+1)!}$$

$$xf'(x) + f(x) = 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \frac{7x^6}{7!} + \ldots \frac{(-1)^n (x^{2n})}{(2n)!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \frac{(-1)^n (x^{2n})}{(2n)!}$$

$$= \cos x$$