2003 BC3

3a. Solve the two equations together to find the point of intersection.

$$\frac{5}{3}y=\sqrt{1+y^2}$$

$$\frac{25}{9}y^2=1+y^2$$

$$\frac{16}{9}y^2=1$$

$$y^2=\frac{9}{16}$$

$$y=\frac{3}{4}$$

$$x=\frac{5}{3}$$

$$\left(\frac{5}{3},\frac{3}{4}\right)$$

3b. $=.3\int_0^{3/4}\left(\sqrt{1+y^2}-\frac{5}{3}y\right)dy=0.346574$

3c.

$$x=r\cos\theta$$

$$y=r\sin\theta$$

$$x^2-y^2=1$$

$$\left(r\cos\theta\right)^2-\left(r\sin\theta\right)^2=1$$

$$r^2(\cos^2\theta-\sin^2\theta)=1$$

$$r^2=\frac{1}{\cos^2\theta-\sin^2\theta}$$

3d.

$$\frac{5}{3}=r\cos\theta$$

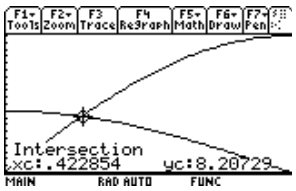
$$\frac{3}{4}=r\sin\theta$$

$$\therefore r=\frac{3}{4\sin\theta}=\frac{5}{3\cos\theta}$$

$$\frac{3}{4\sin\theta}=\frac{5}{3\cos\theta}$$

$$9\cos\theta=20\sin\theta$$

solving these together yields



or

$$\frac{1}{2}\int_0^{0.422854}r^2d\theta$$

$$=\frac{1}{2}\int_0^{0.422854}\frac{1}{\cos^2\theta-\sin^2\theta}d\theta$$