2003 Form B BC6

A. $f(2)=1; f'(2)=\frac{2!}{3}; f''(2)=\frac{3!}{3^2}; f'''(2)=\frac{4!}{3^3}$

$$f(x)=1+\frac{2}{3}(x-2)+\frac{3!}{2!3^2}(x-2)^2+\frac{4!}{3!3^3}(x-2)^3+\ldots+\frac{(n+1)!}{n!3^n}(x-2)^n+\ldots$$

$$f(x)=1+\frac{2}{3}(x-2)+\frac{3}{3^2}(x-2)^2+\frac{4}{3^3}(x-2)^3+\ldots+\frac{(n+1)}{3^n}(x-2)^n+\ldots$$

B. $\lim\limits_{n\to\infty}\left|\dfrac{\frac{n+2}{3^{n+1}}(x-2)^{n+1}}{\frac{n+1}{3^n}(x-2)^n}\right| = \lim\limits_{n\to\infty}\left(\frac{n+2}{n+1}\bullet\frac{1}{3}\right)|x-2|$ Therefore the radius of convergence

$=\frac{1}{3}|x-2|<1 \ \ when \ |x-2|<3$

is 3.

C. The Taylor Series does not converge at x = - 2 because the geometric series only converges for $|x-2|<3$ or -3 < x - 2 < 3 or -1 < x < 5 .