

2003 Form B BC4

A. Velocity Vector:

$$x'(t) = 6e^{3t} - 7e^{-7t}$$

$$y'(t) = 9e^{3t} + 2e^{-2t}$$

So the velocity vector is $\langle 6e^{3t} - 7e^{-7t}, 9e^{3t} + 2e^{-2t} \rangle$

$$\text{speed} = \sqrt{x'(0)^2 + y'(0)^2}$$

$$: = \sqrt{(-1)^2 + (11)^2}$$

$$= \sqrt{122}$$

B.
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$$

$$\lim_{t \rightarrow \infty} \frac{dy}{dx} = \lim_{t \rightarrow \infty} \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \frac{9}{6} = \frac{3}{2}$$

C. To be horizontal $y'(t)=0$, but $y'(t) = 9e^{3t} + 2e^{-2t} > 0$ so there are not values of t where there is a horizontal tangent.

D. For a vertical tangent line we need $x'(t) = 0$ while $y'(t) \neq 0$

$$x'(t) = 6e^{3t} - 7e^{-7t} = 0$$

$$6e^{3t} = 7e^{-7t}$$

$$e^{10t} = \frac{7}{6}$$

$$\ln e^{10t} = \ln \frac{7}{6}$$

$$10t = \ln \frac{7}{6}$$

$$t = \frac{1}{10} \ln \frac{7}{6}$$