2003 Form B BC2

A. Solve each equation for the positive y values:

$$(x-1)^2 + y^2 = 1$$
$$y^2 = 1-(x-1)^2 \quad \textbf{(1)}$$
$$y = \sqrt{1-(x-1)^2}$$

$$x^2 + y^2 = 2$$
$$y^2 = 2 - x^2 \quad \textbf{(2)}$$
$$y = \sqrt{2-x^2}$$

The shaded area is under function 1 between 0 and 1 and under function 2 from 1 to $\sqrt{2}$

$$AREA = \int_0^1 \sqrt{1-(x-1)^2}\,dx + \int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$$

Alternatively the area under function 1 is a quarter of a circle so its area is $\frac{1}{4}\pi 1^2$. So the total area of R could be $AREA = \frac{1}{4}\pi + \int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$

B. To set up the area using dy's we need to first write both equations as functions of y:

$$(x-1)^2 + y^2 = 1$$
$$(x-1)^2 = 1 - y^2$$
$$x = \pm\sqrt{1-y^2} \quad (3)$$
$$x = 1 - \sqrt{1-y^2}$$

$$x^2 + y^2 = 2$$
$$x^2 = 2 - y^2 \quad \textbf{(4)}$$
$$x = \sqrt{2-y^2}$$

Note in solving for the left portion of the circle you must select the negative square root expression. The area is now bounded between these two functions of y from

$$AREA = \int_0^1 \sqrt{2-y^2} - \left(1 - \sqrt{1-y}\right) dy$$

C. $AREA = \int_0^{\pi/4} \frac{1}{2}\left(\sqrt{2}\right)^2 d\theta + \int_{\pi/4}^{\pi/2} \frac{1}{2}\left(2\cos\theta\right)^2 d\theta$