2003 AB6

$$f(x)=\begin{cases}\sqrt{x+1}, & 0\le x\le 3\\ 5-x, & 3<x\le 5\end{cases}$$

6a. $\lim_{x\to 3+} f(x) = 2 = \lim_{x\to 3-} f(x)$ so the function is continuous at x = 3.

6b. The average value has be calculated by finding the area under the function and dividing by the width.

$$f_{avg}(x)=\frac{1}{5}\left(\int_0^3 f(x)dx+\int_3^5 f(x)dx\right)$$

$$=\frac{1}{5}\left(\frac{2}{3}(x+1)^{3/2}\Big|_0^3+5x-\frac{x^2}{2}\Big|_3^5\right)$$

$$=\frac{1}{5}\left(\frac{2}{3}\left((4)^{3/2}-1\right)+\left(\left(25-\frac{25}{2}\right)-\left(15-\frac{9}{2}\right)\right)\right)$$

$$=\frac{1}{5}\left(\frac{2}{3}(7)+\left(\frac{25}{2}-\frac{21}{2}\right)\right)$$

$$=\frac{1}{5}\left(\frac{14}{3}+2\right)$$

$$=\frac{1}{5}\left(\frac{20}{3}\right)$$

$$=\frac{4}{3}$$

6c. Following a similar step to part a yields:

$$\lim_{x\to 3+} f(x) = 3m+2 = 2k = \lim_{x\to 3-} f(x)$$

so $3m+2=2k$ (A)

Since g is differentiable then

$$g'(x)=\begin{cases}\dfrac{k}{2\sqrt{x+1}}, & 0<x<3\\ m, & 3<x<5\end{cases}.$$

The question is do the two derivative parts equal each other as x approaches 3.

For this to be true

$$\frac{k}{2\sqrt{3+1}}=m$$

$$\frac{k}{4}=m \qquad \text{(B)}$$

$$k=4m$$

Solving the two equations A and B together gives

3m+2 = 2(4m)

m = 2/5 k = 8/5