2003 AB4/BC4

4a. A sign study of f ' tells where the function f is increasing and decreasing

x	-3 < x < -2	-2 < x < 0	0 < x < 2	2 < x < 3
f '(x)	positive	negative	negative	negative

So f is increasing between - 3 < x < -2.

4b. A sign study of f " tells where f concavity.

x	-3 < x < 0	0 < x < 2	2 < x <3
f "(x)	negative	positive	negative

So f has a point of inflection at x = 2. At x = 0 the second derivative does not exist.

4c. f ' (0) = -2 so the tangent line is y = -2x + 3.

4d. Looking at the graph you will notice that $\int_{-3}^{0} f'(x)dx = f(x)\Big|_{-3}^{0} = f(0) - f(-3)$

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Then $-1\frac{1}{2} = f(0) - f(-3)$ based on the bounded area from [-3,0].

This leads to $-1\frac{1}{2} = 3 - f(-3)$ since f(0)=3.

Solving yields $4\frac{1}{2} = f(-3)$

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Alternately for this first question you can also find the antiderivative of the straight line, solve for c and then find f(-3) by substitution in the function.

$$f(x) = -\frac{x^2}{2} - 2x + c$$

$$f(0) = 3 = c$$

$$f(x) = -\frac{x^2}{2} - 2x + 3$$

$$f(-3) = 4\frac{1}{2}$$

To find f(4) use the same technique with area:

$$\int_{0}^{4} f'(x)dx = f(x)\Big|_{0}^{4} = f(4) - f(0)$$

$$-8 - (-2\pi) = f(4) - f(0)$$

$$-8 + 2\pi = f(4) - 3$$

$$f(4) = -5 + 2\pi$$